



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

$$\therefore \frac{v^2 x^3}{x^3 - r^3} = 2f(h+d) = \frac{2(1-\delta)(h+d)g}{1+\delta}$$

d can be found by either method in Vol. I, page 134.

202. Proposed by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud, England.

Three equal, uniform, similar rods AB , BC , CD , freely jointed at B and C , are hung from a point by two equal strings attached at A and D . Find the position of equilibrium.

Solution by G. W. GREENWOOD, M. A., Dunbar, Pa.

By symmetry, the strings, of length l , say, make equal angles with the vertical, as do also the rods AB and DC ; denote these angles by θ and ϕ , respectively. The rod BC is horizontal. Denote the length of each rod by a , the weight by w , and the depth of the center of gravity of the system below the point of support by z , the strings being regarded as weightless.

$$\begin{aligned} z &= [w(l \cos \theta + \tfrac{1}{2}a \cos \phi) + w(l \cos \theta + a \cos \phi) + w(l \cos \theta + \tfrac{1}{2}a \cos \phi)] / 3w \\ &= \tfrac{1}{3}[3l \cos \theta + 2a \cos \phi]. \end{aligned}$$

For equilibrium, the value of z must be a maximum.

$$\therefore 0 = 3l \sin \theta d\theta + 2a \sin \phi d\phi \dots (1).$$

Also, by horizontal projection,

$$a = 2l \sin \theta + 2a \sin \phi \dots (2).$$

$$\therefore 0 = l \cos \theta d\theta + a \cos \phi d\phi \dots (3).$$

$\therefore 3 \tan \theta = 2 \tan \phi$ (by eliminating $d\theta$ and $d\phi$ from (1) and (2)). This equation, with equation (3), gives the position of equilibrium.

Also solved by G. B. M. Zerr and J. Scheffer.

AVERAGE AND PROBABILITY.

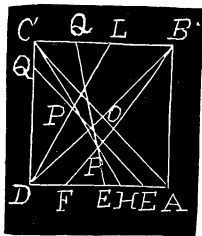
187. Proposed by HENRY HEATON, Belfield, N. D.

Through every point of a given square straight lines are drawn in every possible direction, terminating in the sides of the square. What is the average length of such lines?

II. Solution by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.

Let $ABCD$ be the given square, side a . P the random point coordinates (u, v) . On account of the symmetry of the square we will confine P to the triangle ADC . Let EQ be the random line through P , $m = \tan \theta = \tan QED$. For the area AOD , E must fall on HF to intersect opposite sides and on AH to intersect adjacent sides. For the area COD , E must fall on

HD to intersect opposite sides and on AH to intersect adjacent sides. For opposite sides $EQ = a \operatorname{cosec} \theta$. For adjacent sides $EQ = (a-u) \sec \theta + v \operatorname{cosec} \theta$. For opposite sides, the limits for AOD are, of v , 0 and $\frac{1}{2}a$; of u , $(a-v)$ and



v ; of θ , $-\tan^{-1}\left(\frac{a-v}{u}\right) = \theta_2$ and $\tan^{-1}\left(\frac{a-v}{a-u}\right) = \theta_1$; for COD , the limits of u are $\frac{1}{2}a$ and a ; of v , u and $(a-u)$; of θ , $-\tan^{-1}\left(\frac{v}{a-u}\right) = \theta_3$ and θ_1 . For adjacent sides the limits of u are 0 and a , of v , 0 and u , of θ , $\tan^{-1}\frac{v}{u} = \theta_4$ and θ_1 . Let Δ be the required average length.

Then we have for the denominator:

$$\begin{aligned}
 D &= \int_0^{\frac{1}{2}a} \int_v^{a-v} \int_{\theta_2}^{\theta_1} dv du d\theta + \int_{\frac{1}{2}a}^a \int_{a-u}^u \int_{\theta_3}^{\theta_1} du dv d\theta + \int_0^a \int_0^u \int_{\theta_1}^{\theta_4} du dv d\theta \\
 &= \int_0^{\frac{1}{2}a} \int_v^{a-v} \left[\tan^{-1}\left(\frac{a-v}{a-u}\right) + \tan^{-1}\left(\frac{a-v}{u}\right) \right] dv du \\
 &\quad + \int_{\frac{1}{2}a}^a \int_{a-u}^u \left[\tan^{-1}\left(\frac{a-v}{a-u}\right) + \tan^{-1}\left(\frac{v}{a-u}\right) \right] du dv \\
 &\quad + \int_0^a \int_0^u \left[\tan^{-1}\left(\frac{v}{u}\right) - \tan^{-1}\left(\frac{a-v}{a-u}\right) \right] du dv \\
 &= \int_0^{\frac{1}{2}a} \left[2(a-v) \log \frac{(a-v)\sqrt{2}}{\sqrt{[(a-v)^2 + v^2]}} + \frac{\pi}{2}(a-v) - 2v \tan^{-1}\left(\frac{a-v}{v}\right) \right] dv \\
 &\quad + \int_{\frac{1}{2}a}^a \left[2(a-u) \log \frac{(a-u)\sqrt{2}}{\sqrt{[(a-u)^2 + u^2]}} - \frac{\pi}{2}(a-u) + 2u \tan^{-1}\left(\frac{u}{a-u}\right) \right] du \\
 &\quad + \int_0^a \left[(a-u) \log \frac{\sqrt{[(a-u)^2 + a^2]}}{(a-u)\sqrt{2}} - u \log \sqrt{2} + \frac{\pi a}{4} - a \tan^{-1}\left(\frac{a}{a-u}\right) \right] du \\
 &= \frac{1}{4}a^2 (\pi + 2 \log 2).
 \end{aligned}$$

For the numerator we have

$$\begin{aligned}
 N &= \int_0^{\frac{1}{2}a} \int_v^{a-v} \int_{\theta_2}^{\theta_1} a \operatorname{cosec} \theta dv du d\theta + \int_{\frac{1}{2}a}^a \int_{a-u}^u \int_{\theta_3}^{\theta_1} a \operatorname{cosec} \theta du dv d\theta \\
 &\quad + \int_0^a \int_0^u \int_{\theta_1}^{\theta_4} [(a-u) \sec \theta + v \operatorname{cosec} \theta] du dv d\theta
 \end{aligned}$$

$$\begin{aligned}
&= a \int_0^a \int_v^{a-v} \log \left[\frac{\sqrt{[(a-u)^2 + (a-v)^2]} - (a-u)}{\sqrt{[u^2 + (a-u)^2]} + u} \right] dv \, du \\
&\quad + a \int_a^a \int_{a-v}^u \log \left[\left(\frac{v}{a-v} \right) \left(\frac{\sqrt{[(a-u)^2 + (a-v)^2]} - (a-u)}{\sqrt{[(a-u)^2 + v^2]} + a-u} \right) \right] du \, dv \\
&\quad - \int_0^a \int_0^u \left[(a-u) \log \left(\frac{u}{a-u} \right) \left(\frac{\sqrt{[(a-u)^2 + (a-v)^2]} + (a-v)}{\sqrt{[u^2 + v^2]} + v} \right) \right] \\
&\quad + v \log \left[\left(\frac{v}{a-v} \right) \left(\frac{\sqrt{[(a-u)^2 + (a-v)^2]} - (a-u)}{\sqrt{[u^2 + v^2]} - u} \right) \right] du \, dv \\
&= 2a \int_0^a \left[v \log \left(\frac{\sqrt{[(a-v)^2 + v^2]} + v}{a-v} \right) - (a-v) \log(\sqrt{2}+1) + (a-v) \sqrt{2} \right. \\
&\quad \left. - \sqrt{[(a-v)^2 + v^2]} \right] dv \\
&\quad + 2a \int_a^a \left[2(a-u) \log(\sqrt{2}+1) - (a-u) \log \{ \sqrt{[(a-u)^2 + u^2]} + u \} \right. \\
&\quad \left. - u \log \left(\frac{\sqrt{[(a-u)^2 + u^2]} + (a-u)}{u} \right) + (a-u) \log(a-u) \right] du \\
&\quad + \frac{1}{2} \int_0^a \left[2au - u^2 - a^2 \sqrt{2} + (a-u) \sqrt{[(a-u)^2 + a^2]} \right. \\
&\quad \left. - a^2 \log \left(\frac{\sqrt{[(a-u)^2 + a^2]} - (a-u)}{a(\sqrt{2}-1)} \right) \right] du \\
&= \frac{a^3}{12} [9(\sqrt{2}+1) \log(\sqrt{2}+1) - 4 - 5\sqrt{2}].
\end{aligned}$$

$$\therefore \Delta = \frac{N}{D} = \frac{a}{3} \left(\frac{9(\sqrt{2}+1) \log(\sqrt{2}+1) - 4 - 5\sqrt{2}}{\pi + 2 \log 2} \right)$$

NOTE.—This solution differs from Mr. Heaton's in that Dr. Zerr assumes the lines to be secant lines, that is, lines whose extremities lie on the sides of the square. Mr. Heaton intended the lines to be radial lines, that is, lines the extremities of which are the random point and points on the sides of the square. In both solutions, the same law of distribution has been assumed. ED. F.

188. Proposed by J. EDWARD SANDERS, Reinersville, Ohio.

Find the average length of a hole at random through a given (a) sphere, (b) cube.

Solution by G. B. M., ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.

(a) Let one point remain fixed and be the origin. Then $x^2 + y^2 + z^2 = 2ax = d^2$ is the equation to the sphere of radius a .